

Figure 4: Routing for class 1

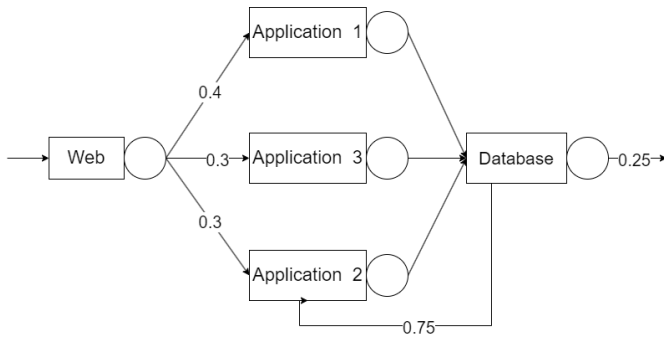


Figure 5: Routing for class 2

As is shown in Table 6, there is only a 2 percent mean relative error between the result of MNA and simulation. 95% of the samples exhibit an ARE within a 5% margin.

5 APPLICATION AND EXAMPLE

We model as a queueing network a three-tier e-commerce system consisting of the following components: *Web Server*: the first point of contact is a web server that handles initial customer requests. *Application Servers*: once the web server processes the initial request, it forwards the customer to one of the three available application servers, depending on their specific needs. *Database Server*: each application server is connected to a central database server responsible for data access.

The system serves two different classes of customers. Upon visiting the website, customers first arrive at the web server. The web server evaluates their needs and routes them to the most appropriate application server for further processing. Once the application server completes its tasks, it interacts with the database server for data storage or retrieval. After being served by the database server, customers have two options: they may continue using the website, in which case they are sent back to the application server for additional services. Alternatively, they may choose to leave the system.

The routing probability of class 1 and class 2 are shown in Figure 4 and Figure 5 respectively. The interarrival time, and the service time are all phase-type distributed. The means of interarrival time of class 1 and class 2 are 4 and 5 respectively, and the SCVs of

Table 7: Service time parameters for real case example

| node | classes | mean | SCV |
|---------------|---------|------|------|
| Application 1 | 1,2 | 0.5 | 1/10 |
| Application 2 | 1 | 1 | 1/10 |
| Application 2 | 2 | 0.3 | 1/10 |
| Application 3 | 1, 2 | 0.5 | 1/10 |
| Database | 1 | 0.3 | 1/10 |
| Database | 2 | 0.5 | 1/10 |

interarrival time are all 1/10. The means and SCVs of the service time are generated as shown in Table 7. For this example, QNA provides a result with a relative error of 8.6 percent while MNA provides a more accurate result with a relative error of 1.5 percent.

6 CONCLUSION

In this paper, we have proposed MNA, a new algorithm for solving multiclass PH queueing networks that leverages the matrix-analytic method. The method has been shown to improve the accuracy of the class QNA algorithm.

In future work, we seek to integrate additional features into MNA. In particular, the method should be extended to incorporate functionalities such as self-loops, class-switching, and mixed workloads. A comparison with gradient-based methods to seek the fixed point would also be beneficial.

Additionally, throughput calculation for closed multi-class queueing networks may face challenges due to its use of fixed iterations, which may not converge. A more efficient and accurate method, perhaps based on gradient search, may be needed.

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