Towards Evaluation/Mitigation Risk of Systemic Failures in a Recoverable Network with Redistributed Elastic Load

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ABSTRACT

While systemic failure/overload in recoverable networks with load redistribution is a common phenomenon, current ability to evaluate and moreover mitigate the corresponding systemic risk is vastly insufficient due to complexity of the problem and relying on oversimplified models. The proposed framework in this paper for systemic risk evaluation relies on approximate dimension reduction at the onset of systemic failure. Assuming a general failure/recovery microscopic model, the macro-level system dynamics is approximated by a 2-state Markov process alternating between systemically operational and failed states.

ACM Reference format:
Vladimir Marbukh. 2023. Towards Evaluation/Mitigation Risk of Systemic Failures in a Recoverable Network with Redistributed Elastic Load. In *the Companion of the 2023 ACM/SPEC International Conference on Performance Engineering (ICPE'23 Companion), April 15–19, 2023, Coimbra, Portugal.* ACM, New York, NY, USA. 2 pages. https://doi.org/10.1145/3578245.3584926

1 INTRODUCTION

While explosive growth in scale and interconnectivity of resource sharing systems of various nature is driven by economics, these benefits are inherently associated with risk of a systemic failure [1]-[2]. Prohibitively high complexity of accounting for interdependencies of component failures due to load redistribution from failed to operational components creates inherent tradeoff between model accuracy and tractability [2]-[3]. This tradeoff explains currently existing focus on modelling effects of specific aspects of the problem, e.g., topology, failure/recovery mechanisms, etc. However, complicated interactions between these aspects make current understanding of the corresponding performance/risk tradeoffs severely insufficient. This paper suggests an approximate, yet accurate, "macroscopic" description of the onset of systemic failure, which incorporates "microscopic" details of the failure propagation process. We hope that the proposed macroscopic model will reveal "universal" features of systemic failures in large-scale networked systems of various nature with different failure/recovery mechanisms. This hope is based on analogy between systemic failures and phase transitions in physics, where such universality exists [4].

ICPE '23 Companion, April 15–19, 2023, Coimbra, Portugal.

ACM ISBN 979-8-4007-0072-9/23/04. https://doi.org/10.1145/3578245.3584926

We assume a very general microscopic failure/recovery model described by a Markov process with locally interacting components and driven by Network Utility Maximization (NUM) [5] for reallocation of elastic load. This microscopic model covers a broad range of applications, including communication and transportation networks, IoT, etc. The macro-level failure dynamics is approximated by a two-state Markov process which alternates between systemically operational and failed states. Transition rates of this macro-process are derived from micro-level dynamics under approximation of Landau theory of phase transitions [6]. Sections 2 describes failure propagation at micro-level. Section 3 derives macro-level failure/recovery dynamics and evaluates the systemic risk.

2 MICRO-LEVEL DYNAMICS

Consider network with N nodes and L bi-directional links $l = 1,..,L$ with variable capacities $c_l(t) = \hat{c}_l\delta_l(t)$, where $\delta_l(t) = 1$ if link l is operational at moment t and $\delta_l(t) = 0$ otherwise. Link l fails at link congestion-dependent rate λ_l and recovers at fixed rate μ_l, where congestion levels are defined below. We assume that each node $n = 1,..,N$ has utility $u_{nk}(x)$ of obtaining point-to-point bandwidth to node $k \in \{1,..,N\}\backslash n$, where functions $u_{nk}(x)$ are increasing, strictly concave and differentiable in $x \geq 0$. Given $\delta = (\delta_l)$, bandwidth allocation maximizes the aggregate utility [5]:

$$W(\delta) = max_{x_{r\geq 0}} \sum_{(n,k)} u_{nk}\left(\sum_{r\in R_{nk}} x_r\right) \quad (1)$$

subject to the aggregate load on link l:

$$\sum_{r:l\in r} x_r \leq (1-\delta_l)\hat{c}_l, \quad (2)$$

where R_{nk} is the set of feasible routes between nodes n and k. Maximization (1)-(2) occurs much faster than link failures/recoveries. Dual to optimization problem (1)-(2) is

$$min_{\gamma\geq 0} max_{x\geq 0} \mathcal{L}(x,\gamma|\delta), \quad (3)$$

where Lagrangian is

$$\mathcal{L} = \sum_{(n,k)} u_{nk}\left(\sum_{r\in R_{nk}} x_r\right) - \sum_l \gamma_l[\sum_{r:l\in r} x_r - (1-\delta_l)\hat{c}_l] \quad (4)$$

and $\gamma = (\gamma_l)$ is a vector of Lagrange multipliers [5]. Since Largange multipliers γ_l characterize congestion on links l, we assume that link l failure rate is an increasing and concave function of γ_l: $\lambda_l = \lambda_l(\gamma_l)$. We also assume that different link failures/recoveries are jointly statistically independent.

Due to assumed time scale separation, evolving vector $\delta(t) = (\delta_l(t)) \in \{0,1\}^{|L|}$ can be approximated by a Markov process with a large number $|L|$ of locally interacting components, where $|L|$ is the number of links in the system. Given system state at moment $t \geq 0$, $\delta(t) = \delta \in \{0,1\}^{|L|}$, component δ_l rate of transition $0 \to 1$ is $\lambda_l[\gamma_l(\delta_{-l})]$, where $\delta_{-l} = \left(\delta_j(t), j \neq l\right)$. This

rate $\lambda_l[\gamma_l(\delta_{-l})]$ depends only on the "neighboring" components δ_j where $\forall j \in r \subseteq R_{nk}$ and all pair of nodes (n,k) such that feasible routes R_{nk} include link l. Apparently, rate $\lambda_l[\gamma_l(\delta_{-l})]$ is an increasing function of vector δ_{-l}. Component δ_l transition $1 \to 0$ rate μ_l is independent of vector δ_{-l}. Probability distribution of Markov process $\delta(t)$, $P(t,\delta)$ is determined by differential Kolmogorov equations. Under some natural assumptions, process $\delta(t)$ has unique steady-state distribution $P(\delta) = lim_{t\to\infty} P\,(t,\delta)$ which is determined by the corresponding steady-state Kolmogorov equations. The fundamental problem is prohibitively high dimension $2^{|L|}$ of this system. Even more restrictive is that evaluation of process $\delta(t)$ transition rates $\lambda_l[\gamma_l(\delta_{-l})]$ requires solving $2^{|L|}$ problems (3)-(4) for each $\delta \in \{0,1\}^{|L|}$.

3 RISK OF SYSTEMIC FAILURE

Consider mean-field approximation which is based on simplifying assumption that operational statuses of different links are jointly statistically independent:

$$P(\delta) \approx \prod_{l=1}^{|L|}\left[\tilde{\delta}_l^{\delta_l}(1-\tilde{\delta}_l)^{1-\delta_l}\right]. \tag{5}$$

Under this approximation, probability of link l failure is

$$\tilde{\delta}_l = \lambda_l(\tilde{\gamma}_l)/[\mu_l + \lambda_l(\tilde{\gamma}_l)], \tag{6}$$

where $\lambda_l(\tilde{\gamma}_l)$ is "effective" failure rate, Lagrange multipliers $\tilde{\gamma} = (\tilde{\gamma}_l)$ is determined by solution to the corresponding dual (4):

$$(\tilde{x},\tilde{\gamma}) = argmin_{\gamma} max_x \mathcal{L}(x,\gamma|\tilde{\delta}). \tag{7}$$

Mean-field equations (6)-(7) form a closed system of $2|L| + |R|$ non-linear fixed-point equations, where $|R|$ is the total number of feasible routes in the system. Assuming that system (6)-(7) has an "operational" asymptotically stable equilibrium $\tilde{\delta}^* = (\tilde{\delta}_l^*)$ characterized by "low" link failure probabilities $\tilde{\delta}_l^* \ll 1$, $l = 1,..,L$, systemic failure may be associated with existence of another asymptotic equilibrium $\tilde{\delta}^{**} = (\tilde{\delta}_l^{**})$ characterized by "much higher " failure probabilities for some links $\tilde{\delta}_l^{**}$ and "much lower" aggregate utility (1): $W(\tilde{\delta}^{**}) \ll W(\tilde{\delta}^*)$. Due to prohibitively high dimension of system (6)-(7) we follow [6] for further dimension reduction close to the point where equilibrium $\tilde{\delta}^*$ loses stability. Consider expansion of link failure rates:

$$\lambda_l[\gamma_l(\delta_{-l})] = \lambda_l[\gamma_l(0)] + \sum_{i\neq l}\lambda_l\left[\gamma_l\left(\delta^{(i)}\right)\right]\delta_i + \cdots, \tag{8}$$

where $\lambda_l[\gamma_l(0)] \ll 1$, binary vectors $\delta^{(0)} = 0$, $\delta^{(i)} = (\chi_{il}, l = 1,..,L)$, $i = 0,1,..,L$, and Kronecker symbol $\chi_{il} = 1$ if $i = l$ and $\chi_{il} = 0$. Substituting (8) into right-hand size of (6) gives us the following system of $|L|$ linear fixed-point equations:

$$\tilde{\delta}_l = \lambda_l[\gamma_l(0)]/\mu_l + \sum_{i\neq l}\left(\lambda_l\left[\gamma_i\left(\delta^{(i)}\right)\right]/\mu_i\right)\tilde{\delta}_l. \tag{9}$$

Dimension reduction [6], inspired by Landau theory of phase transitions, is based on Perron-Frobenius (P-F) theory of non-negative matrix $A = (A_{ij})_{i,j=1}^L$ with components $A_{ij} = \lambda_i\left[\gamma_j\left(\delta^{(j)}\right)\right]/\mu_i$. Contagion-free region is given by condition that P-F eigenvalue of matrix A, Γ is less than one: $\Gamma < 1$. Operational equilibrium $\tilde{\delta}^*$ looses stability along leading eigenvalue of matrix $A = (A_{ij})_{i,j=1}^L$, $\xi = (\xi_l)_{l=1}^L$, which allowed for approximating transitions between two equilibria $\tilde{\delta}^*$ and $\tilde{\delta}^{**}$ by a one-dimensional birth-death Markov process. Time-scale separation between these "rare" transitions and typical time scale of Markov microprocess $\delta(t)$ allows us to approximate these transitions in "slow" time scale by a 2-state Markov process: $\tilde{\pi}(t) \in \{0,1\}$, where states $\tilde{\pi}(t) = 0$ and $\tilde{\pi}(t) = 1$ correspond to systemically operational and failed states respectively, and transition rates $\tilde{\lambda}: 0 \to 1$ and $\tilde{\mu}: 1 \to 0$ are determined by the failure/recovery rates of individual links. The utility loss averaged over given time horizon of interest $[0,T]$ is

$$Loss^{[0,T]} = T^{-1}\left[W(\tilde{\delta}^*) - W(\tilde{\delta}^{**})\right]\int_0^T \pi(t)dt. \tag{10}$$

Since $Loss^{[0,T]}$ is a random variable, we identify the risk of systemic failure with some established risk measure associated with random loss (10). In particular, the corresponding Value at Risk is

$$VaR_{1-\alpha}^{[0,T]} = inf\{x: P\left(Loss^{[0,T]} > x\right) \leq \alpha\}, \tag{11}$$

where $0 < \alpha < 1$ characterizes systemic risk tolerance.

Figure 1 plots VaR (11) vs. normalized, i.e., adjusted for the time horizon of interest, risk averseness $\theta := -\tilde{\lambda}T/\ln(1-\alpha)$ in natural asymptotic regime $\tilde{\lambda} \downarrow 0$, $T \uparrow \infty$, $\tilde{\lambda}T = O(1)$. Cases $\alpha \uparrow 1 \Rightarrow \theta \downarrow 0$ and $\alpha \downarrow 0 \Rightarrow \theta \uparrow \infty$ correspond to extremely low and respectively high normalized risk averseness.

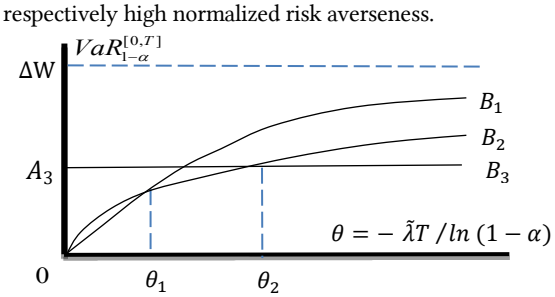

Figure 1. Value at Risk vs. normalized risk averseness

Curves $0B_1$, $0B_2$, and A_3B_3 correspond to different scenarios of system resource allocation in robustness, i.e., contagion prevention, on the one hand, and recoverability on the other hand, given the total amount of these resources. In all scenarios, $VaR_{1-\alpha}^{[0,T]}$ increases with θ and upper bounded by $\Delta W = W(\tilde{\delta}^*) - W(\tilde{\delta}^{**})$. Curves $0B_1$, $0B_2$, (A_3B_3) correspond to discontinuous (continuous) emergence of systemically failed equilibrium [4]. System resource allocation, which minimizes $VaR_{1-\alpha}^{[0,T]}$, depends on θ: for $0 \leq \theta < \theta_1$. $\theta_1 < \theta \leq \theta_2$ or $\theta_2 < \theta$ the optimal allocation corresponds to curves $0B_1, 0B_2$ or A_3B_3 respectively.

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